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Higher Dimensional Unified Description of Early Universe with Variable G and Λ

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Abstract A homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model with varying gravitational and cosmological constant is studied in the context of higher dimensional space time. Exact solution of the field equations are obtained by using the "gamma law" equation of state $p=(\gamma-1)\rho$, where γ is adiabatic parameter varies continuously as the universe expands. The functional form γ which is assumed to be the function of scale factor R as proposed by Carvalho (1996) is used to analyse the behavior of scale factor R, cosmological constant Λ and the gravitational constant G for two different phases: inflation and radiation. The various physical aspects of the early cosmological models has also been discussed in the framework of higher dimensional space time.

Keywords Higher dimensional space time · Cosmology · Early universe · Inflationary model · Cosmological parameters

1 Introduction

It is widely believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimensions beyond those four observed so far. The absences of any signature of extra dimension in current experiments is usually explained the compactness of extra dimensions. The idea of dimensional reduction or self compactification fits in particularly well in cosmology because if we believe in the big bang, our universe was much smaller at the early stage and the present four dimensional stage

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could have been preceded by a higher dimensional one (Chodos and Detweiler [1]). A number of authors [2–4] have studied the physics of the universe in higher dimensional space time. In this work we consider $(n+2)$-dimensional Robertson Walker (RW) model as a test case. In FRW type of homogeneous cosmological model, the dimensionality has a marked effect on the time temperature relation of the universe and our universe appears to cool more slowly in higher dimensional space time (Chatterjee [5]). In recent year, models with a relic cosmological constant Λ have received considerable attention among researchers for various reasons (see Refs. [6–11] and references therein). We should realize that the existence of a nonzero cosmological constant in Einstein's equations is a feature of deep and profound consequence. The recent observations indicate that $\Lambda \sim 10^{-55}$ cm^{-2} while particle physics prediction for Λ is greater than this value by a factor of order 10^{120}. This discrepancy is known as cosmological constant problem. Some of the recent discussions on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant are investigated by Ratra and Peebles [12], Dolgov [13–15], Sahni and Starobinsky [16], Padmanabhan [17] and Peebles [18]. For earlier reviews on this topic, the reader is referred to Zeldovich [19], Weinberg [20] and Carroll et al. [21].

Recent observations of type Ia supernovae (SNe Ia) at redshift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be *accelerating*, behavior attributed to "dark energy" with negative pressure. These observations (Perlmutter et al. [22–24]; Riess et al. [25, 26]; Garnavich et al. [27, 28]; Schmidt et al. [29]) strongly favour a significant and positive value of Λ. The main conclusion of these observations is that the expansion of the universe is accelerating.

A number of authors have argued in favour of the dependence $\Lambda \sim t^{-2}$ first expressed by Bertolami [30, 31] and later on by several authors [32–45] in different context. Recently, motivated by dimensional grounds in keeping with quantum cosmology, Chen and Wu [46], Abdel-Rahaman [47, 48] considered a Λ varying as R^{-2}. Carvalho et al. [49], Waga [50], Silveira and Waga [51], Vishwakarma [52] have also considered/modified the same kind of variation. Such a dependence alleviates some problems in reconciling observational data with the inflationary universe scenario. Al-Rawaf and Taha and Al-Rawaf [53, 54] and Overdin and Cooperstock [55] proposed a cosmological model with a cosmological constant of the form $\Lambda = \beta\frac{\ddot{R}}{R}$, where β is a constant. Following the same decay law recently Arbab [56, 57] have investigated cosmic acceleration with positive cosmological constant and also analyse the implication of a model built-in cosmological constant for four-dimensional space time. The cosmological consequences of this decay law are very attractive. This law provides reasonable solutions to the cosmological puzzles presently known. One of the motivations for introducing Λ term is to reconcile the age parameter and the density parameter of the universe with recent observational data. Vishwakarma [58] has studied the magnitude-red-shift relation for the type Ia supernovae data and the angular size-red-shift relation for the updated compact radio sources data Gurvits [59] by considering four variable Λ-models: $\Lambda \sim R^{-2}$, $\Lambda \sim H^{-2}$, $\Lambda \sim \rho$ and $\Lambda \sim t^{-2}$.

Carvalho [60], Singh [61, 62] have studied Robertson Walker model in four dimension general theory of relativity by using equation of state $p = (\gamma - 1)\rho$, where the adiabatic γ varies with cosmic time. A unified description of early universe has been presented by him in which an inflationary period is followed by a radiation dominated period. Singh and Sriram [63], Patra et al. [64] and Singh et al. [65] have studied some aspects of early universe within the frame work of four dimensional alternative theories of gravitation. The work of the above authors motivate to consider further work in some modified theories.

In this paper we have consider a spatially homogeneous and isotropic higher dimensional FRW cosmological model with varying gravitational constant G and cosmological constant Λ. Our approach is phenomenological similar in some aspect to that of Carvalho [60], Singh [61, 62] in the context of higher dimensional space time. The gamma law of equation of state is applied in which the parameter gamma depends on the scale factor R. The unified description of early evolution of universe is presented with variable G and Λ in which inflationary phase is followed by a radiation dominated phase. The various physical aspect of model are also discussed in context of higher dimensional space time. We observe that the solutions reduce to the Singh [61] results for $n = 2$.

2 Field Equations

We consider the spatially homogeneous and FRW higher dimensional line element of the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 dX_n^2\right], \tag{1}$$

where $R(t)$ is the scale factor, $k = 0, \pm 1$ is the curvature parameter and

$$dX_n^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \sin^2\theta_2 \cdots \sin^2\theta_{n-1} d\theta_n^2.$$

Einstein's field equations with gravitational and time dependent cosmological constants read as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G(t) T_{\mu\nu} + \Lambda(t) g_{\mu\nu}, \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor, $G(t)$ and $\Lambda(t)$ being the variable gravitational and cosmological constants. The energy momentum tensor $T_{\mu\nu}$ in the presence of perfect fluid has the form

$$T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho) u_\mu u_\nu, \tag{3}$$

where p and ρ are respectively the energy density and pressure of cosmic fluid, and u_μ is the $(n+2)$ velocity vector such as $u_\mu u^\mu = 1$. The divergence of the (2) gives

$$(8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu})^{;\,\nu} = 0. \tag{4}$$

Using co-moving coordinates

$$u_\mu = (1, 0, 0, \ldots, (n+1)\text{times}). \tag{5}$$

Einstein's field equations (2) and (3) for the metric (1) take the form

$$8\pi G(t)\rho = \frac{n(n+1)}{2}\left[\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right] - \Lambda(t), \tag{6}$$

$$8\pi G(t) p = -\frac{n\ddot{R}}{R} - \frac{n(n-1)}{2}\left[\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right] + \Lambda(t). \tag{7}$$

In the uniform cosmology $G = G(t)$ and $\Lambda = \Lambda(t)$ so that the conservation (4) is given by

$$\dot{\Lambda} = -8\pi \dot{G}\rho, \tag{8}$$

where dot $(\cdot)$ denotes differentiation with respect to t.

In terms of Hubbel parameter $H = \dot{R}/R$ (6) and (7) can be written as

$$\dot{H} + H^2 = -\frac{8\pi G(t)}{n(n+1)}(p(n+1) + \rho(n-1)) + \frac{2\Lambda(t)}{n(n+1)}, \tag{9}$$

and

$$H^2 = \frac{16\pi G(t)}{n(n+1)}\rho + \frac{2\Lambda(t)}{n(n+1)} - \frac{k}{R^2}. \tag{10}$$

To solved the system of (8–10) by assuming the equation of state

$$p = (\gamma - 1)\rho, \tag{11}$$

where γ is an adiabatic parameter. In many cases the value of this parameter taken to constant ($0 \leq \gamma \leq 2$). A large number of models has been proposed to study the universe in this range of adiabatic parameter. Carvalho [4] studied the Roberson Walker models with zero curvature in four dimensional general relativity by using the gamma-law equation state (11) where γ depends upon the scale factor $R(T)$ as

$$\gamma(R) = \frac{4}{3}\left[\frac{A(\frac{R}{R_*})^2 + (\frac{a}{2})(\frac{R}{R_*})^a}{A(\frac{R}{R_*})^2 + (\frac{R}{R^*})^a}\right], \tag{12}$$

where A is the constant and a is free parameter to the power of the cosmic time lies in $0 \leq a < 1$. The maximum value for the inflationary phase is $a = 1$. Here R_* is certain reference value such that for $R \ll R_*$, inflationary phase ($\gamma = \frac{2a}{3}$) of evolution of the universe and for $R \gg R_*$ we have a radiation dominated phase ($\gamma = \frac{4}{3}$) .

In this paper in the context of higher dimensional space time we can assumed the functional form of γ as

$$\gamma(R) = \frac{(n+2)}{(n+1)}\left[\frac{A(\frac{R}{R_*})^2 + (\frac{a}{2})(\frac{R}{R_*})^a}{A(\frac{R}{R_*})^2 + (\frac{R}{R^*})^a}\right]. \tag{13}$$

Here R_* is certain reference value such that for $R \ll R_*$, inflationary phase ($\gamma = \frac{na}{(n+1)}$) of evolution of the universe and for $R \gg R_*$ we have a radiation dominated phase ($\gamma = \frac{(n+2)}{(n+1)}$). Using (11) into (9), we obtain

$$\dot{H} + H^2 = -\frac{8\pi G(t)}{n(n+1)}[\gamma(n+1) - 2]\rho + \frac{2\Lambda(t)}{n(n+1)}. \tag{14}$$

By eliminating ρ from (8) and (14) for $k = 0$ (flat universe), we get

$$\dot{H} + H^2 = \frac{2}{n(n+1)}\left[\gamma\frac{(n+1)}{2} - 1\right]G(t)\frac{\dot{\Lambda}}{\dot{G}} + \frac{2}{n(n+1)}\Lambda(t). \tag{15}$$

Equation (15) can also be rewrite in the form

$$HH^{'} + \frac{H^2}{R} = \frac{2}{n(n+1)}\left[\frac{\gamma(n+1)}{2} - 1\right]\frac{G(t)}{R}\frac{\Lambda^{'}}{G^{'}} + \frac{2}{n(n+1)}\frac{\Lambda(t)}{R}, \tag{16}$$

where a prime ($'$) denotes differentiations with respect to R.

3 Solution of the Field Equation

We obtain the solution of (16) by assuming the following form of $G(t)$ and $\Lambda(t)$:

3.1 Case (i)

We assume

$$G(t) = \alpha H, \tag{17}$$

and

$$\Lambda(t) = \beta H^2, \tag{18}$$

where α and β are dimensionless positive constant. From (17) and (18), (16) can be expressed as

$$H^{'} + \frac{H}{R}\left[\frac{n(n+1)+2\beta}{n(n+1)} - \beta\gamma\right] = 0. \tag{19}$$

Using value of $\gamma(R)$ from (13) into (19) and integrating, we get

$$H = \frac{C}{R^{\frac{n(n+1)+2\beta}{n(n+1)}}[A(\frac{R}{R_*})^2 + (\frac{R}{R_*})^a]^{-\frac{\beta(n+2)}{2(n+1)}}}, \tag{20}$$

where C is the constant of integration. If $H = H_*$ for $R = R_*$, we have the relation between constant of integration C and A as

$$C = H_* R_*^{\frac{n(n+1)+2\beta}{n(n+1)}} (1+A)^{-\frac{\beta(n+2)}{2(n+1)}}. \tag{21}$$

By using (21) into (20) and after integration obtain the expression for t in term of the scale factor R, is given by

$$H_* R_*^{\frac{n(n+1)+2\beta}{n(n+1)}} (1+A)^{-\frac{\beta(n+2)}{2(n+1)}} t = \int R^{\frac{2\beta}{n(n+1)}}\left[A\left(\frac{R}{R_*}\right)^2 + \left(\frac{R}{R_*}\right)^a\right]^{-\frac{\beta(n+2)}{2(n+1)}} dR. \tag{22}$$

The deceleration parameter can be defined as $q = \frac{-R\ddot{R}}{\dot{R}^2}$, it follows from (19) that during the course of evolution, is given by

$$q = \frac{\beta[2 - n(n+1)\gamma]}{n(n+1)}, \tag{23}$$

which clearly depends upon R via γ. We know solve (22) in two different phases of the universe: (i) Inflationary phase; (ii) Radiation dominated phase.

3.1.1 Inflationary Phase ($R \ll R_*$)

For the inflationary phase ($R \ll R_*$), the second term on the right hand side of (22) dominates which gives the solution for the scale factor R ($a \neq 0$) as

$$R = R_*\left(\left[\frac{\beta(4 - n(n+2)a) + 2n(n+1)}{2n(n+1)}\right]\frac{H_*}{(1+A)^{\frac{\beta(n+2)}{2(n+1)}}} t\right)^{\frac{2n(n+1)}{\beta(4-n(n+2)a)+2n(n+1)}}. \tag{24}$$

From (24) we observe that during the inflation, the dimension of the universe increases according to

$$R \propto t^{\alpha_0}, \tag{25}$$

where

$$\alpha_0 = \frac{2n(n+1)}{\beta[4 - n(n+2)a] + 2n(n+1)},$$

which is the case of power law inflation and for ($\beta = 0$) we see that the radius of the curvature increases linearly with the age of the universe. From (24) we find the following physical parameters:

$$H = \alpha_0 t^{-1}, \tag{26}$$

$$G = \alpha\alpha_0 t^{-1}, \tag{27}$$

$$\Lambda = \beta\alpha_0^2 t^{-2}, \tag{28}$$

$$\rho = \frac{1}{16\pi\alpha}[(n(n+1) - 2\beta)]\alpha_0 t^{-1}. \tag{29}$$

From the above physical parameter we observe that for the physical significance the energy density $\rho > 0$, and gravitational constant $G > 0$, we must have $0 < \beta < n(n+1)/2$ and the energy density ρ decreases with cosmic time increases and it tends to zero as t tends to infinity. We also observe that the spatial volume is zero at $t = 0$. The expanding model has singularity at $t = 0$. For $\beta = 0$ the Hubble parameter $H = t^{-1}$ and the gravitational constant $G \propto t^{-1}$ and $\rho \propto t^{-1}$ as the age of the universe, whereas the $\Lambda = 0$, the radius of the universe increases linearly with cosmic time. This form of Λ is physically reasonable as the observation suggest that Λ is very small in the present universe. The asymptotic value of the deceleration parameter in the limit $\frac{R}{R_*} \ll 1$, is given in (23) by $q = \frac{2\beta(1-n^2a/2)}{n(n+1)}$ for the limiting value $\gamma = na/(n+1)$.

3.1.2 Radiation Dominated Phase ($R \gg R_*$)

For radiation dominated phase ($R \gg R_*$), the first term on the right-hand side of (22) dominates which gives the solution for the scale factor R ($a \neq 0$) as

$$R = R_*\left[\left(1 - \frac{\beta n(n+2) - 2\beta}{n(n+1)}\right)\left(\frac{A}{1+A}\right)^{\frac{\beta(n+2)}{2(n+1)}} H_* t\right]^{\frac{1}{1-\frac{\beta n(n+2)-2\beta}{n(n+1)}}}. \tag{30}$$

From (30) we observe that during the radiation dominated phase the dimension of the universe increase in higher dimension space time according to

$$R \propto t^{b_0}, \tag{31}$$

where

$$b_0 = \left[1 - \frac{\beta n(n+2) - 2\beta}{n(n+1)}\right]^{-1},$$

which is the case of power law expansion for the expanding universe. For $\beta = 0$, the radius of the universe linearly with cosmic time. From (30) we find the following solution for

Hubble parameter, gravitational constant, cosmological constant and energy density

$$H = b_0 t^{-1}, \tag{32}$$

$$G = \alpha b_0 t^{-1}, \tag{33}$$

$$\Lambda = \beta b_0 t^{-2}, \tag{34}$$

$$\rho = \frac{1}{16\pi\alpha}(n(n+1) - 2\beta) b_0 t^{-1}. \tag{35}$$

For the energy density $\rho > 0$ we must have $0 < \beta < \frac{n(n+1)}{2}$. We observe that the spatial volume is zero at $t = 0$. The expanding model has singularity at $t = 0$. For $\beta = 0$, it is also observed that the Hubble parameter $H = t^{-1}$ and the gravitational constant $G \propto t^{-1}$ an $\rho \propto t^{-1}$ as the age of the universe, whereas the $\Lambda = 0$. By putting the limiting value $\gamma = (n+2)/(n+1)$ in (23), the asymptotic value of the deceleration parameter in the limit $\frac{R}{R_*} \gg 1$, is given by

$$q = \beta \left[\frac{2 - n(n+2)}{n(n+1)} \right]. \tag{36}$$

This shows that the deceleration parameter is constant.

3.2 Case (ii)

Here we assume gravitational constant $G(t)$ is inversely proportional to Hubble parameter H i.e.

$$G = \frac{\alpha_0}{H}. \tag{37}$$

By using (37) and the same cosmological constant Λ defined in (18), (16) can be expressed as

$$H^{'} + \left[\left(1 - \frac{6\beta}{n(n+1)} \right) + \frac{2\gamma\beta}{n} \right] \frac{H}{R} = 0. \tag{38}$$

After integration we get

$$H = \frac{C_1}{R^{(1-\frac{6\beta}{n(n+1)})} [A(\frac{R}{R_*})^2 + (\frac{R}{R_*})^a]^{\frac{\beta(n+2)}{2(n+1)}}}, \tag{39}$$

where C_1 is the constant of integration. An expression of the t in term of the scale factor R is given by

$$H_* R^{(1-\frac{6\beta}{n(n+1)})} (1+A)^{\frac{\beta(n+2)}{2(n+1)}} t = \int R^{-\frac{6\beta}{n(n+1)}} \left[A\left(\frac{R}{R_*}\right)^2 + \left(\frac{R}{R_*}\right)^a \right]^{\frac{\beta(n+2)}{2(n+1)}} dR. \tag{40}$$

From (40) we obtain the solutions for inflationary as well as radiation dominated phases as follows:

3.2.1 Inflationary Phase

$$R \propto t^{c_0}, \tag{41}$$

$$H \propto t^{-1}, \tag{42}$$

$$G \propto t, \tag{43}$$

$$\Lambda \propto t^{-2}, \tag{44}$$

$$\rho \propto t^{-3}, \tag{45}$$

where $c_0 = \frac{2n(n+1)}{\beta[an(n+2)-12]+2n(n+1)}$.

3.2.2 Radiation Dominated Phase

$$R \propto t^{d_0} \tag{46}$$

$$H \propto t^{-1}, \tag{47}$$

$$G \propto t, \tag{48}$$

$$\Lambda \propto t^{-2}, \tag{49}$$

$$\rho \propto t^{-3}, \tag{50}$$

where $d_0 = \frac{n(n+1)}{\beta[n(n+2)-6]+n(n+1)}$.

From the above solutions we observe that the energy density ρ varies inversely as the cube of the cosmic time and hence $\rho \to \infty$ as $t \to 0$. The expanding model has singularity at $t = 0$. We have also observe that the gravitational constant G increases with the cosmic time in both the phases whereas the cosmological constant Λ varies inversely as the square of the cosmic time.

The density parameter of the universe Ω_{rad} is given by

$$\Omega_{\text{rad}} = \frac{16\pi\rho}{n(n+1)H^2} = 1 - e_0, \tag{51}$$

The density parameter due to vacuum contribution is defined as

$$\Omega_\Lambda = \frac{2\Lambda}{n(n+1)H^2} = e_0, \tag{52}$$

where $e_0 = 2\beta/n(n+1)$.

From (51) and (52), we obtain

$$\Omega_{\text{rad}} + \Omega_\Lambda = 1. \tag{53}$$

According to high red-shift supernovae and CMB, the preliminary results from the advancing field of cosmology suggest that the universe may be an accelerating universe with a dominant contribution to its energy density in the form of the cosmological constant Λ term. The results, when combined with CMB anisotropy observations on intermediate angular scales, strongly support a flat universe:

$$\Omega_{\text{rad}} + \Omega_\Lambda = 1.$$

This results can be obtained in any case of inflationary and radiation dominated phases.

4 Concluding Remark

In this work we have presented the higher dimensional cosmological solutions for spatially homogeneous and isotropic FRW model with varying cosmological and gravitational constant. The obtained solutions generalise to higher dimension the well know results in the four dimensional space time. It is observed that the difference is significant at least in the principle to the analogous situation in four dimensional space time. The solutions reduce to the four dimensional form when $n = 2$. We have taken the gamma law of equation of state (13) as a function of scale factor in the context of higher dimensional space time and obtained the solution with the assumption $G \propto H$, $\Lambda \propto H^2$ and $G \propto H^{-1}$. For $a = 0$ the universe is infinitely old, since $R \to 0$ as $t \to -\infty$, for $R \ll R_*$. For $R \gg R_*$, it enters into the radiation dominated phase. For $a \leq 0$, γ slowly increases from $\frac{na}{(n+1)}$ to $\frac{n+2}{n+1}$, when $R \gg R_*$. The period of the evolution is described by inflationary phase followed by radiation dominated phase. The constant G and Λ are allowed to depend on the cosmic time t. In case (i) it has been observed that: $\rho \propto t^{-1}$, $G \propto t^{-1}$ and $\Lambda \propto t^{-2}$. The cosmological parameter retains the natural dimension with time (i.e. $\Lambda \propto t^{-2}$). The physical parameter have the values in $0 < \beta < \frac{n(n+1)}{2}$. In case (ii) we have observed that $\rho \propto t^{-3}$, $G \propto t^{-1}$ and $\Lambda \propto t^{-2}$ for both inflation and radiation dominated phases. It is also shown that if the universe is spatially flat, the total energy density parameter ($\Omega_{\text{total}} = \Omega_{\text{rad}} + \Omega_\Lambda$) gives unity. This implies that the cosmological constant supplies the "missing matter" requires to make $\Omega_{\text{total}} = 1$ as suggested by the inflationary models though on the basis of little observational evidence. The expanding universe has singular at $t = 0$. In this way the unified description of early evolution of the universe is possible with variables G and Λ in the framework of higher dimensional space time.

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